Torsion: theory and possible observables

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Abstract.[2] We discuss the theoretical basis for the search of the possible experimental manifestations of the torsion field at low energies. First, the quantum field theory in an external gravitational field with torsion is reviewed. The renormalizability requires the nonminimal interaction of torsion with spinor and scalar (Higgs) fields. The Pauli-like equation contains new torsion-dependent terms which have a different structure as compared with the standard electromagnetic ones. The same concerns the nonrelativistic equations for spin-1/2 particle in an external torsion and electromagnetic fields. Second, we discuss the propagating torsion. For the Dirac spinor coupled to the electromagnetic and torsion field there is some additional softly broken local symmetry associated with torsion. As a consequence of this symmetry, in the framework of effective field theory, the torsion action is fixed with accuracy to the values of the coupling constant of the torsion-spinor interaction, mass of the torsion and higher derivative terms. The introduction of the Higgs field spoils the consistency of this scheme, and the effective quantum field theory for torsion embedded into the Standard Model is not possible. The phenomenological consequences of the torsion-fermion interaction are drown and the case of the torsion mass of the Planck order is discussed.

1. Introduction

Gravity with torsion and especially the interaction of torsion with a spinor field has attracted attention for a long time [7, 8, 9, 13]. In a last years interest in theories with torsion has increased because of the success of the formal development of string theory [10] which is (together with its modifications and generalizations) nowadays regarded as the main candidate for the unique description of all quantum fields. String theory predicts that the set of fields should be extended in such a way that, along with the known fundamental interactions, some new ones appear. In particular, one of these is the completely antisymmetric torsion field which is an independent quantity serving, along with the metric, for the description of space-time. Probably this was the reason why in recent years there has been an increasing interest in possible physical effects related to torsion. Another motivation to study the gravity with torsion is that it appears naturally as a compensating field for a local gauge transformations [11] (see also [12] for a review of this approach and further references). Recently there were several interesting developments about the possible manifestations of torsion (see, for example, [3, 15, 14, 16, 5, 17]). Most of these works discuss the effects of classical or quantum matter fields on an external torsion background.

If the starting point of our consideration is string theory, torsion should be accociated with the stress-tensor for the antisymmetric tensor which shows up in the string effective action. In this case, since the massive parameter of expansion $1/\alpha'$ in string theory is of the Planck order, the only case when the propagating torsion can be seen, is the cancellation

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of massive term in the string effective action. Of course, there is no guarantee that this cancellation doesn't take place in some unknown version of string theory. On the other hand, if we suppose that such a cancellation doesn't take place, then the mass of torsion is of the Planck order and the effective field theory approach does not apply. From physical point of view this means that torsion doest exist as an independent field, but only as some excitation. After the phase transitions which breaks the string into individual field, torsion should disappear because of its huge mass, which provides extremely fast damping with time. The only possibility is, perhaps, that the constant component of the torsion should not be subject of such a dumping and, after being weakened in the inflationary epoch, can exist as a very weak constant (in our part of the Universe) 4-pseudovector. Thus the discussion of the possible torsion effects should be concentrated on the study of possibilities for the propagating torsion and, independently, for the effects of constant background torsion to the classical and quantum dynamics of the matter particles and fields.

The study of the renormalization of quantum field theory in an external gravitational field with torsion [1] (see also [2, 4]) has shown that in the gauge theories like QED, the Standard Model or GUT's, the interaction of matter fields with torsion has special features. With scalars torsion can interact in a nonminimal way only. On the other hand, if one introduces the interactions between spin-1/2, spin-0 and spin-1 fields, the renormalizability requires nonminimal interaction with both spinors and scalars [2]. Thus we arrive at the necessity of introducing nonminimal interaction between the Dirac field and torsion, and we have to introduce some new nonminimal parameter – a charge, which characterizes such an interaction.

Probably the most simple way to understand the special features of torsion is to study nonrelativistic limit of the Dirac equation – that is on the generalization of the Pauli equation for the case of an external electromagnetic and torsion fields. Here we are going to review the derivation of this approximation and also obtain the next to the leading order corrections in the framework of the Foldy-Wouthuysen transformation. We also establish some properties of the higher order corrections, and demonstrate some global symmetry which holds for the Dirac spinor in external electromagnetic and torsion fields.

In the second part of this review the propagating torsion is discussed. We prove that the action of dynamical torsion necessary contains massive vector field and to evaluate its possible observational consequences. The action for the torsion pseudovector is derived. This action depends on two free parameters (one of them is torsion mass). The study the phenomenological consequences of this action, therefore, reduces to the search of the upper bounds for these two parameters from the modern (in our case high-energy) experiments. As the result of this study we obtain these bounds on the parameters of the torsion action using the known data from particle physics.

The paper is organized in the following way. In the next section we introduce basic notations, and give a very brief review of gravity with torsion. In the next sections we comment on the renormalization and renormalization group for the gauge theories in an external gravitational field with torsion. Some additional symmetry which holds for the spinor field coupled to torsion is established. In section 4 the details of the Foldy–Wouthuysen transformation are presented. The equations of motion for spinning particles are discussed in section 5 and section 6 is devoted to the effective quantum field theory for the propagating torsion.

2. The background notions for the gravity with torsion

In the space - time with independent metric $g_{\mu\nu}$ and torsion $T^{\alpha}_{\ \beta\gamma}$ the connection $\tilde{\Gamma}^{\alpha}_{\ \beta\gamma}$ is nonsymmetric, and

$$\tilde{\Gamma}^{\alpha}_{\ \beta\gamma} - \tilde{\Gamma}^{\alpha}_{\ \gamma\beta} = T^{\alpha}_{\ \beta\gamma} \tag{1}$$

If one introduces the metricity condition $\tilde{\nabla}_{\mu} g_{\alpha\beta} = 0$ where the covariant derivative $\tilde{\nabla}_{\mu}$ is constructed on the base of $\tilde{\Gamma}^{\alpha}_{\ \beta\gamma}$, then the following solution for connection $\tilde{\Gamma}^{\alpha}_{\ \beta\gamma}$ can be easily found

$$\tilde{\Gamma}^{\alpha}_{\ \beta\gamma} = \Gamma^{\alpha}_{\ \beta\gamma} + K^{\alpha}_{\ \beta\gamma} \tag{2}$$

where $\Gamma^{\alpha}_{\ \beta\gamma}$ is standard symmetric Christoffel symbol and $K^{\alpha}_{\ \beta\gamma}$ is contorsion tensor

$$K^{\alpha}_{\ \beta\gamma} = \frac{1}{2}\left(T^{\alpha}_{\ \beta\gamma} - T_{\beta\ \gamma}^{\ \alpha} - T_{\gamma\ \beta}^{\ \alpha}\right) \tag{3}$$

It is convenient to divide the torsion field into three irreducible components which are: the trace $T_{\beta} = T^{\alpha}_{\ \beta\alpha}$, the pseudotrace $S^{\nu} = \varepsilon^{\alpha\beta\mu\nu} T_{\alpha\beta\mu}$ and the tensor $q^{\alpha}_{\ \beta\gamma}$, where the last satisfies two conditions

$$q^{\alpha}_{\ \beta\alpha} = 0, \qquad \varepsilon^{\alpha\beta\mu\nu} q_{\alpha\beta\mu} = 0$$

Then the torsion field can be written in the form

$$T_{\alpha\beta\mu} = \frac{1}{3}\left(T_{\beta}\, g_{\alpha\mu} - T_{\mu}\, g_{\alpha\beta}\right) - \frac{1}{6}\varepsilon_{\alpha\beta\mu\nu} S^{\nu} + q_{\alpha\beta\mu} \tag{4}$$

The curvature with torsion is constructed as usual $[\tilde{\nabla}_{\mu}, \tilde{\nabla}_{\nu}]\Phi^{A} = \tilde{R}^{A}_{B\,\mu\nu}\Phi^{B}$ and it has obvious relation with the Riemannian curvature $\tilde{R}^{A}_{B\,\mu\nu} = R^{A}{}_{B\,\mu\nu} + torsion\ terms.$

Consider the Dirac field ψ in an external gravitational field with torsion. The standard way to introduce the minimal interaction with external fields is the substitution of the partial derivatives ∂_{μ} by the covariant ones.

The covariant derivatives of the spinor field ψ are defined as

$$\tilde{\nabla}_{\mu}\psi = \partial_{\mu}\psi + \frac{i}{2}\tilde{w}_{\mu}^{\ ab}\sigma_{ab}\psi$$

$$\tilde{\nabla}_{\mu}\bar{\psi} = \partial_{\mu}\bar{\psi} - \frac{i}{2}\tilde{w}_{\mu}^{\ ab}\bar{\psi}\sigma_{ab} \tag{5}$$

where $\tilde{w}_{\mu}^{\ ab}$ are the components of spinor connection. We use standard representation for the Dirac matrices.

$$\beta = \gamma^{0} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \qquad \alpha = \gamma^{0}\gamma = \begin{pmatrix} 0 & \sigma \\ \sigma & 0 \end{pmatrix}$$

$$\gamma_{5} = \gamma^{0}\gamma^{1}\gamma^{2}\gamma^{3}, \quad \sigma_{a\,b} = \frac{i}{2}(\gamma_{a}\gamma_{b} - \gamma_{b}\gamma_{a})$$

The verbein e^{a}_{μ} obeys the equations $e^{a}_{\mu}e_{\nu a} = g_{\mu\nu}$, $e^{a}_{\mu}e^{\mu b} = \eta^{ab}$ where η^{ab} is the Minkowsky metric. The gamma matrices in curved space - time are introduced as $\gamma^{\mu} = e^{\mu}_{a}\gamma^{a}$ and obviously satisfy the metricity condition $\tilde{\nabla}_{\mu}\gamma^{\beta} = 0$. The condition of metricity enables one to find the explicit expression for spinor connection which agrees with (2).

$$\tilde{w}_{\mu}^{\ ab} = \frac{1}{4}(e^{b}_{\nu}\partial_{\mu}e^{\nu a} - e^{a}_{\nu}\partial_{\mu}e^{\nu b}) + \bar{\Gamma}^{\alpha}_{\ \nu\mu}(e^{\nu a}e^{b}_{\alpha} - e^{\nu b}e^{a}_{\alpha}) \tag{6}$$

Then the action of spinor field minimally coupled with torsion is written the form

$$S = \int d^4x \, \sqrt{-g} \, \{ \frac{i}{2} \bar{\psi} \gamma^\mu \tilde{\nabla}_\mu \psi - \frac{i}{2} \tilde{\nabla}_\mu \bar{\psi} \gamma^\mu \psi + m \bar{\psi} \psi \} \tag{7}$$

where m is the mass of the Dirac field and The expression (7) can be rewritten in the form

$$S = \int d^4x \, \sqrt{-g} \, \{ i \bar{\psi} \gamma^\mu (\nabla_\mu + \frac{i}{8} \gamma_5 S_\mu) \psi + m \bar{\psi} \psi \} \tag{8}$$

where ∇ is Riemann covariant derivative (without torsion).

One can see that the spinor field minimally interacts only with the pseudovector S_μ part of the torsion tensor. The nonminimal interaction is more complicated. On the dimensional grounds one can introduce the nonminimal coupling of the form

$$S = \int d^4x \, \sqrt{-g} \, \{ i \bar{\psi} \gamma^\mu (\partial_\mu + i \eta_1 \gamma_5 S_\mu + i \eta_2 T_\mu) \psi + m \bar{\psi} \psi \} \tag{9}$$

Here η_1, η_2 are the dimensionless parameters of the nonminimal coupling of spinor fields with torsion. The minimal interaction corresponds to the values $\eta_1 = \frac{1}{8}, \;\; \eta_2 = 0$.

The introduction of the nonminimal interaction seems to be artificial since on the classical level one can explain the use of a nonminimal action only as an attempt to explore the most general case. However the situation is different in quantum region where the nonminimal interaction is necessary condition of consistency of the theory. The reason is the following. It is well-known that the interaction of quantum fields leads to the divergences and therefore the renormalization is needed. The requirement of the multiplicative renormalizability makes us to introduce the nonminimal interaction of torsion with spinor and scalar fields.

With the scalar field φ torsion may interact only nonminimally, because $\tilde{\nabla} \varphi = \partial \varphi$. The action of free scalar field including the nonminimal interaction with antisymmetric torsion has the form

$$S_{sc} = \int d^4x \, \sqrt{-g} \, \{ \frac{1}{2} g^{\mu\nu} \, \partial_\mu \varphi \, \partial_\nu \varphi + \frac{1}{2} m^2 \varphi^2 + \frac{1}{2} \xi_i P_i \varphi^2 \} \tag{10}$$

where ξ_i are new nonminimal parameters and

$$P_1 = R, \qquad P_2 = \nabla_\alpha T^\alpha, \qquad P_3 = T_\alpha T^\alpha, \qquad P_4 = S_\alpha S^\alpha, \qquad P_3 = q_{\alpha\beta\gamma} q^{\alpha\beta\gamma} \,. \tag{11}$$

We accept that the gauge vector field does not interact with torsion at all, because such an interaction, generally, contradicts to the gauge invariance. This can be easily seen from the relation

$$\tilde{\nabla}_\mu A_\nu - \tilde{\nabla}_\nu A_\mu = \partial_\mu A_\nu - \partial_\nu A_\mu + K^\lambda{}_{\mu\nu} \, A_\lambda . \tag{12}$$

The nonminimal interaction with abelian vector field may be indeed implemented in the form of the surface term

$$S_{n-m,vec} \; = \; i \, \alpha \int d^4x \, \epsilon^{\alpha\beta\gamma\sigma} \, F_{\alpha\beta} \, S_{\gamma\sigma} . \tag{13}$$

Other nonminimal terms are also possible for the general torsion but they are relevant only for the nonzero T_μ and $q_{\alpha\beta\gamma}$ components of the torsion tensor and thus we are not interested in them.

3. Renormalization in curved space-time with torsion

The renormalization of quantum field theories in curved space-time with torsion has been discussed in full details in the book [4], and therefore there are no reasons to reproduce these details here. Thus we only establish the main qualitative results which will prove important in what follows.

If the quantum theory contains scalar and spinor fields linked by the Yukawa interaction $h\varphi\bar{\psi}\psi$, then the nonminimal parameters η_1, xi_4 are necessary for the renormalizability, because there are diagrams which lead to the corresponding divergences. As a result the nonminimal parameters become effective charges and their running with scale is governed by the corresponding renormalization group equations. For the quantum field theory in the external torsion field the renormalization of the parameters η_1, ξ_4 is universal. In particular, the β-function for the nonminimal parameter η_1 has the form

$$\beta_{\eta_1} = \frac{C}{(4\pi)^2}\, h^2\, \eta_1 \,, \tag{14}$$

where model-dependent C is always positive.

Despite the nonminimal fermion action contains two dimensionless nonminimal parameters η_1, η_2 we shall use only one of them and put $\eta_2 = 0$. Reasons: (i) The minimal interaction includes only η_1 term, therefore only η_1 as an essential parameter. (ii) The η_2-term looks very similar to the electromagnetic interactions. and it can be revoked by simple redefinition of the variables and constants. (iii) The string-induced action depends on the completely antisymmetric torsion which is equivalent to S_μ , so one needs only η_1, ξ_4 and ξ_1. Below we denote $\eta_1 = \eta$ and always take $\eta_2 = 0$.

4. Equation for spinor field in the nonrelativistic approximation

Adding usual electromagnetic interaction to the spinor field action we get:

$$i\hbar\frac{\partial\psi}{\partial t} = \{c\alpha\mathbf{p} - e\alpha\mathbf{A} - \eta_1\alpha\mathbf{S}\gamma_5 + e\Phi + \eta_1\gamma_5 S_0 + mc^2\beta\}\psi \tag{15}$$

Here the dimensional constants $\hbar$ and c are taken into account, and we divide $A_\mu = (\Phi, \mathbf{A})$, $S_\mu = (S_0, \mathbf{S})$. Following standard procedure one has to write

$$\psi = \begin{pmatrix}\varphi\\ \chi\end{pmatrix} exp(\frac{imc^2t}{\hbar})\,, \tag{16}$$

that gives

$$(i\hbar\frac{\partial}{\partial t} - \eta_1\sigma\cdot\mathbf{S} - e\Phi)\varphi = (c\sigma\cdot\mathbf{p} - e\sigma\cdot\mathbf{A} - \eta_1 S_0)\chi \tag{17a}$$

and

$$(i\hbar\frac{\partial}{\partial t} - \eta_1\sigma\cdot\mathbf{S} - e\Phi + 2mc^2)\chi = (c\sigma\cdot\mathbf{p} - e\sigma\cdot\mathbf{A} - \eta_1 S_0)\varphi \tag{17b}\,.$$

Within the nonrelativistic approximation $\chi \ll \varphi$. Now we keep only the term $2mc^2\chi$ in the left side of (17a) and then it is possible to find χ from (17b). In the leading order in $\frac{1}{c}$ we meet the following equation for φ.

$$i\hbar\frac{\partial\varphi}{\partial t} = \{\eta_1\sigma\cdot\mathbf{S} + e\Phi + \frac{1}{2mc^2\chi}(c\sigma\cdot\mathbf{p} - e\sigma\cdot\mathbf{A} - \eta_1 S_0)^2\}\varphi\,. \tag{18}$$

The last can be easily rewritten in the Schroedinger form

$$i\hbar\frac{\partial\varphi}{\partial t} = \hat{H}\varphi\,, \tag{19}$$

where the Hamiltonian has the form [3]

$$\hat{H} = \frac{1}{2m}\pi^2 + B_0 + \sigma\cdot\mathbf{Q}\,,$$

$$\pi = \mathbf{P} - \frac{e}{c}\mathbf{A} - \frac{\eta_1}{c}\sigma S_0\,,$$

$$B_0 = e\Phi - \frac{1}{mc^2}\eta_1^2 S_0^2\,,$$

$$\mathbf{Q} = \eta_1\mathbf{S} + \frac{\hbar\, e}{2mc}\mathbf{H}\,. \tag{20}$$

Here $\mathbf{H} = rot\mathbf{A}$ is the magnetic field strength. This equation is the analog of the Pauli equation in the more general case of an external torsion and electromagnetic fields. Thus, there is a big difference between the torsion and the electromagnetic terms. For instance, the term $-\frac{1}{mc}\eta_1 S_0\mathbf{p}\cdot\sigma$ does not have the analogies in quantum electrodynamics.

A little bit more complicated approach comes from the *Foldy-Wouthuysen transformation* [5]. The initial Hamiltonian has the form:

$$H = \beta m + \mathcal{E} + \mathcal{G} \tag{21}$$

where $\mathcal{E} = e\,A_0 - \eta\,\gamma_5\,\alpha\,\mathbf{S}\,,\quad \mathcal{G} = \alpha\;(\mathbf{p} - e\mathbf{A}) + \eta\,\gamma_5\,S_0$ are the even and odd parts of the Hamiltonian, and we have used the units $c = \hbar = 1$.

Our purpose is to find a unitary transformation which separates "small" and "large" components of the Dirac spinor. One can easily see that $\mathcal{E}$ and $\mathcal{G}$ given above obey the relations $\mathcal{E}\,\beta = \beta\,\mathcal{E}\,,\ \mathcal{G}\,\beta = -\beta\,\mathcal{G}$ and therefore one can safely use the standard prescription for the lowest-order approximation for $\mathcal{S}$: $\mathcal{S} = -\frac{i}{2m}\,\beta\,\mathcal{G}$. Thus

$$H' = e^{i\mathcal{S}}\;(H - i\,\partial_t)\;e^{-i\mathcal{S}} \tag{}$$

where $\mathcal{S}$ has to be chosen in an appropriate way. To find $\mathcal{S}$ and H' in a form of the nonrelativistic expansion, one has to take $\mathcal{S}$ of order $1/m$. Then the standard result is

$$H' = H + i\;[\mathcal{S},H] - \frac{1}{2}\;[\mathcal{S},[\mathcal{S},H]] - \frac{i}{6}\;[\mathcal{S},[\mathcal{S},[\mathcal{S},H]]]$$

$$+\frac{1}{24}\;[\mathcal{S},[\mathcal{S},[\mathcal{S},[\mathcal{S},H]]]] - \dot{\mathcal{S}} - \frac{i}{2}\;\left[\mathcal{S},\dot{\mathcal{S}}\right] + \frac{1}{6}\;\left[\mathcal{S},\left[\mathcal{S},\dot{\mathcal{S}}\right]\right] + ... \tag{22}$$

Now we take

$$H' = \beta m + \mathcal{E}' + \mathcal{G}' \tag{23}$$

where $\mathcal{G}'$ is of order $1/m$, and one has to perform second FW transform with $\mathcal{S}' = -\frac{i}{2m}\,\beta\mathcal{G}'$. This leads to the

$$H'' = \beta m + \mathcal{E}' + \mathcal{G}'' \tag{24}$$

with $\mathcal{G}'' \approx 1/m^2$.

The third FW with $\mathcal{S}'' = -\frac{i}{2m}\beta\mathcal{G}''$ removes odd operators in the given order of the nonrelativistic expansion, so that we finally obtain the usual result

$$H''' = \beta(m + \frac{1}{2m}\mathcal{G}^2 - \frac{1}{8m^3}\,\mathcal{G}^4) + \mathcal{E} - \frac{1}{8m^2}\left[\mathcal{G},\,([\mathcal{G},\mathcal{E}] + i\dot{\mathcal{G}})\right]$$

Substituting here $\mathcal{E}$ and $\mathcal{G}$ after some algebra we arrive at the final form of the Hamiltonian

$$\begin{aligned} H''' = \beta\left[m + \frac{1}{2m}\left(\mathbf{p} - e\mathbf{A} + \eta S_0\sigma\right)^2 - \frac{1}{8m^3}\,\mathbf{p}^4\right] + eA_0 - \\ -\eta\left(\sigma\cdot\mathbf{S}\right) - \frac{e}{2m}\sigma\cdot\mathbf{B} - \frac{\eta^2}{m}\,\beta S_0^2 - \\ -\frac{e}{8m^2}\left[\nabla\mathbf{E} + i\sigma\cdot(\nabla\times\mathbf{E}) + 2\sigma\cdot\,(\mathbf{E}\times\mathbf{p})\right] + \\ +\frac{\eta}{8m^2}\left[-\sigma\cdot\nabla\dot{S}_0 + \{p_i,\{p^i,(\sigma\cdot\mathbf{S})\}\}-\right. \\ -2\,(\nabla\times\mathbf{S})\cdot\mathbf{p} + 2i\,(\sigma\cdot\nabla)\,(\mathbf{S}\cdot\mathbf{p}) + 2i\,(\nabla\mathbf{S})\,\,(\sigma\cdot\mathbf{p})\} \end{aligned} \tag{25}$$

Here the approximation is as usual for the electromagnetic case; that is we keep terms $(\textit{kinetic energy})^3$ and $(\textit{kinetic energy})^2\cdot(\textit{potential energy})$.

One can indeed proceed in this way and get separated Hamiltonian with any given accuracy in $1/m$. We remark that the first five terms of (25) reproduce the Pauli-like equation with torsion.

5. The motion of spin-$1/2$ particle on an external torsion and electromagnetic background

Let us start from the Pauli-like equation.

$$H = \frac{1}{2m}\pi^2 + B_0 + \sigma\cdot\mathbf{Q} \tag{26}$$

where $\pi, B_0, \mathbf{Q}$ are defined in (16) and $\pi = m\mathbf{v}$ and $\mathbf{v} = \dot{\mathbf{x}}$ is the velocity of the particle. The expression for the canonical conjugated momenta $\mathbf{p}$ is

$$\mathbf{p} = m\mathbf{v} + \frac{e}{c}\mathbf{A} + \frac{\eta_1}{c}\sigma S_0$$

and σ is the coordinate corresponding to spin.

Let us now introduce the operators of $\hat{x}_i$, $\hat{p}_i$, spin $\hat{\sigma}_i$ and input the equal - time commutation relations:

$$[\hat{x}_i,\hat{p}_j] = i\hbar\delta_{ij}, \qquad [\hat{x}_i,\hat{\sigma}_j] = [\hat{p}_i,\hat{\sigma}_j] = 0, \qquad [\hat{\sigma}_i,\hat{\sigma}_j] = 2i\varepsilon_{ijk}\hat{\sigma}_k$$

The Hamiltonian operator $\bar{H}$ is easily constructed in terms of the operators $\hat{x}_i, \hat{p}_i, \hat{\sigma}_i$ and then these operators yield the equations of motion

$$i\hbar\frac{d\hat{x}_i}{dt} = [\hat{x}_i, H]\,, \quad i\hbar\frac{d\hat{p}_i}{dt} = [\hat{p}_i, H]\,, \quad i\hbar\frac{d\hat{\sigma}_i}{dt} = [\hat{\sigma}_i, H]\ . \tag{27}$$

After deriving the commutators in (27) we arrive at the explicit form of the operator equations of motion. Now we can omit all the terms which vanish when $\hbar \to 0$.

$$\frac{d\mathbf{x}}{dt} = \frac{1}{m}\left(\mathbf{p} - \frac{e}{c}\mathbf{A} - \frac{\eta_1}{c}\sigma S_0\right) = \mathbf{v}, \tag{28a}$$

$$\frac{d\mathbf{v}}{dt} = e\mathbf{E} + \frac{e}{c}\left[\mathbf{v}\times\mathbf{H}\right] - \eta_1\left(\sigma\cdot\nabla\right)\mathbf{S} - \eta_1\left[\sigma\times\nabla\times\mathbf{S}\right] - \frac{\eta_1}{c}\sigma\,\frac{\partial S_0}{\partial t} - \frac{\eta_1}{c}S_0\frac{d\sigma}{dt} +$$

$$+\frac{\eta_1}{c}\{(\mathbf{v}\cdot\sigma)\,\nabla S_0 - (\mathbf{v}\cdot\nabla S_0)\,\sigma\} + \frac{1}{mc^2}\,\eta_1^2\,\nabla(S_0^2)\,, \tag{28b}$$

$$\frac{d\sigma}{dt} = \left[\mathbf{R}\times\sigma\right]$$

$$\mathbf{R} = \frac{2\eta_1}{\hbar}\left[\mathbf{S} - \frac{1}{c}\mathbf{v}S_0\right] + \frac{e}{mc}\mathbf{H} \tag{28c}$$

These are the (quasi)classical equations of motion for the particle in an external torsion and electromagnetic fields. The operator arrangement problem is irrelevant because of the use of $\hbar \to 0$ limit.

The equations (28) contain some terms which have a qualitatively new structure. Thus we see that the standard claim concerning magnetic field analogy of torsion effects is not completely correct, and there exist serious difference between magnetic field and torsion.

Consider some solutions of the equations of motion of a spinning particle in a space with torsion but without electromagnetic field. For the cosmological reasons we are mainly interested in the cases of constant axial vector $S_\mu = (S_0, \mathbf{S})$. In this case the equations have the form:

$$\frac{d\mathbf{v}}{dt} = -\eta\,\mathbf{S}\,(\mathbf{v}\cdot\sigma) - \frac{\eta S_0}{c}\,\frac{d\sigma}{dt}\,,$$

$$\frac{d\sigma}{dt} = +\frac{2\eta}{\hbar}\,\left[\mathbf{S}\times\sigma\right] - \frac{2\eta S_0}{\hbar c}\,\left[\mathbf{v}\times\sigma\right]\,. \tag{29}$$

Consider first the case $S_0 = 0$ so that only $\mathbf{S}$ is present. Since $\mathbf{S} = const$, we can safely put $S_{1,2} = 0$. The solution for spin can be easily found to be $\sigma_3 = \sigma_{30} = const$ and

$$\sigma_1 = \rho\,\cos\left(\frac{2\eta S_3 t}{\hbar}\right), \qquad \sigma_2 = \rho\,\sin\left(\frac{2\eta S_3 t}{\hbar}\right) \tag{30}$$

where $\rho = \sqrt{\sigma_{10}^2 + \sigma_{20}^2}$. For the velocity we have $v_1 = v_{10} = const$, $v_2 = v_{20} = const$, In case $\sigma_3 = 0$ one finds oscillating solution and for $\sigma_3 \neq 0$ the solution is

$$v_3(t) = \left[\,v_{30} + \frac{\rho\hbar\,\left(\sigma_3 v_{10}\hbar - 2mv_{20}\right)}{4m^2 + \hbar^2\sigma_3^2}\right]\,e^{-\frac{\eta S_3\sigma_3}{m}\,t} -$$

$$-\frac{\rho\hbar}{4m^2 + \hbar^2\sigma_3^2}\,\left[A_1\cos\left(\frac{2\eta S_3 t}{\hbar}\right) + A_2\sin\left(\frac{2\eta S_3 t}{\hbar}\right)\right] \tag{31}$$

with $A_1 = \sigma_3 v_{10}\hbar - 2mv_{20}$ and $A_2 = \sigma_3 v_{20}\hbar + 2mv_{10}$.

Thus one meets

(i) precession of the spin around the direction of $\mathbf{S}$

(ii) oscillation of the particle velocity in this same direction is accompanied (for $\sigma_3 \neq 0$) by the exponential damping of the initial velocity in this direction.

Consider another special case $\mathbf{S} = 0$, which is the form of the torsion field motivated by isotropic cosmological models (see, for example, [4] and references there). Then the equations of motion have a form

$$\frac{d\mathbf{v}}{dt} = -\frac{\eta S_0}{c}\frac{d\sigma}{dt} = \frac{2\eta^2 S_0^2}{c\hbar}\,[\mathbf{v}\times\sigma] \tag{32}$$

Despite the fact that those equations formally look like a nonlinear system of equations for six unknowns, they can be integrated immediately if we notice that the time variations of the variables do not affect the vector product. Hence the general solutions are

$$\mathbf{v}(t) = \mathbf{v}_0 + [\mathbf{v}_0\times\sigma_0]\,\frac{2\eta^2 S_0^2}{c\hbar}\,t\,,$$
$$\sigma(t) = \sigma_0 - [\mathbf{v}_0\times\sigma_0]\,\frac{2\eta S_0}{\hbar}\,t\,. \tag{33}$$

Thus the motion of such a particle is a motion with constant acceleration. This is possible, in the presence of torsion, for electrically neutral particles with spin.

Some remark is in order. The torsion field is supposed to act on the spin of particles but not on their angular momentum. Therefore a motion like the one described above will occur for individual electrons or other particles with spin as well as for macroscopic bodies with a nonzero overall spin orientation. However it does not occur for the (charged or neutral) bodies with a random orientation of spins.

6. Effective quantum field theory and propagating torsion

Independent on the development of the classical and quantum field theory in an external torsion field it is important to establish the form of the action for the torsion itself and to study the possible experimental effects of dynamical torsion. There can be very different approaches to the construction of the torsion action (see, for example, [11, 23]). We shall consider the construction of the effective quantum field theory [19] for dynamical (propagating) torsion, and establish the torsion action using the consistency of this theory as a criterion. In this case the consistency conditions include unitarity of the S-matrix and the gauge-invariant renormalizability, but not the power-counting renormalizability. The procedure of formulating of effective quantum field theory for the new type of interaction looks as follows:

i) One has to establish the field content of the dynamical torsion theory and the form of interactions between torsion and other fields.

ii) It is necessary to take into account the symmetries of this interactions and formulate the action in such a way that the resulting theory is unitary and renormalizable as an effective field theory.

Indeed there is no guarantee that these requirements are consistent with each other, but the inconsistency may only indicate that some symmetries are lost or that the consistent theory with the given particle content is impossible. For simplicity we consider below only flat metric.

Let us start from the action of the fermion coupled to vector and torsion

$$S_{1/2} = i\int d^4x\,\bar{\psi}\,[\,\gamma^\alpha\,(\partial_\alpha - ieA_\alpha + i\,\eta\,\gamma_5\,S_\alpha\,) - im\,]\,\psi \tag{34}$$

In case of the action (34) there are two gauge symmetries, and second of them is softly broken.

$$\psi' = \psi\, e^{\alpha(x)}, \qquad \bar{\psi}' = \bar{\psi}\, e^{-\alpha(x)}, \qquad A'_\mu = A_\mu - e^{-1}\,\partial_\mu\alpha(x)$$
$$\psi' = \psi\, e^{\gamma_5\beta(x)}, \qquad \bar{\psi}' = \bar{\psi}\, e^{\gamma_5\beta(x)}, \qquad S'_\mu = S_\mu - \eta^{-1}\,\partial_\mu\beta(x)$$

This symmetry structure enables one to derive the unique form of the torsion action. The higher derivative terms of the action are not seen at low energies. Thus one arrives at the expression:

$$S_{tor} = \int d^4 \,\Big\{ -a\, S_{\mu\nu}S^{\mu\nu} + b\,(\partial_\mu S^\mu)^2 + M_{ts}^2\, S_\mu S^\mu \Big\}\,, \tag{37}$$

where $S_{\mu\nu} = \partial_\mu S_\nu - \partial_\mu S_\nu$ and a, b are some positive parameters. (37) contains both transversal vector mode and the scalar mode[3]. In the $a = 0$ case only the scalar mode, and for $b = 0$ only the vector mode propagate.

In the unitary theory of the vector field the longitudinal and transversal modes can not propagate simultaneously [22], and therefore one has to choose one of the parameters a, b to be zero. In fact the only correct choice is $b = 0$, because the symmetry (36), which is spoiled by the massive terms only, is preserved in the renormalization of the massless sector. This structure of renormalization is essentially the same as for the Yang-Mills theories with spontaneous symmetry breaking. On the dimensional grounds the gauge invariant counterterm $\int S_{\mu\nu}^2$ has to appear if we take the loop corrections into account. We remark that the $\int S_{\mu\nu}^2$-term must be included into the action even if the tree-level effects are evaluated, if only such consideration is regarded as an approximation to any reasonable quantum theory. Thus the torsion action is given by

$$S_{tor} = \int d^4x\, \Big\{ -\frac{1}{4}\, S_{\mu\nu}S^{\mu\nu} + M_{ts}^2\, S_\mu S^\mu \Big\}\,. \tag{38}$$

In the last expression we put the conventional coefficient $-1/4$ in front of the kinetic term.

In order to illustrate that the kinetic counterterm with $b = 0$ and the massive counterterm really show up, let us perform a simple derivation of the divergences coming from the fermion loops. The divergent part of the one-loop effective action is given by the expression

$$\Gamma_{div}[A, S] = -\,\mathrm{Tr}\,\ln \hat{H}\,\big|_{div}\,,$$

where

$$\hat{H} = i\gamma^\alpha\,(\mathcal{D}_\alpha - im) \quad \text{and} \quad \mathcal{D}_\alpha = \partial_\alpha - ieA_\alpha + i\eta\gamma_5\, S_\alpha\,. \tag{39}$$

In order to calculate this functional determinant one can make the transformation which preserves covariance with respect to the derivative $\mathcal{D}_\alpha$ [18].

$$\mathrm{Tr}\,\ln \hat{H} = \frac{1}{2}\,\mathrm{Tr}\,\ln i\gamma^\alpha\,(\mathcal{D}_\alpha - im)\,\cdot\, i\gamma^\beta\,(\mathcal{D}_\beta + im) = \mathrm{Tr}\,\ln\Big(-\hat{1}\Box + \hat{\Pi}\Big)\,,$$

where $\hat{\Pi} = \sigma^{\mu\nu}F_{\mu\nu}$. Calculating, using the standard Schwinger-DeWitt technique, we arrive at the counterterms

$$\Delta S[A_\mu, S_\alpha] = \frac{\mu^{D-4}}{\varepsilon}\int d^Dx\,\Big\{\frac{2e^2}{3}F_{\mu\nu}F^{\mu\nu} + \frac{2\eta^2}{3}S_{\mu\nu}S^{\mu\nu} + 8m^2\eta^2 S^\mu S_\mu\Big\}$$

[3] This case has been considered, from different points of view, in [20, 21].

where $\varepsilon = (4\pi)^2\,(D-4)$ is parameter of the dimensional regularization. The form of the counterterms is in perfect agreement with the above consideration based on the symmetry transformation (36). Namely, the one-loop divergences contain $S_{\mu\nu}^2$ and the massive term while the $(\partial_\nu S^\nu)^2$ term is absent. One has to notice that the topological counterterm

$$ie\eta \int d^4x\, \epsilon^{\alpha\beta\mu\nu} S_{\mu\nu} F_{\alpha\beta}$$

cancels within the covariant scheme. This terms appears with a nonzero coefficient within the noncovariant schemes. This means, in fact, the cancellation of the one-loop contribution to the axial (or gauge) anomaly. According to the Adler-Bardeed theorem in this case the anomaly is absent at higher loops too. The appearance of anomaly at one loop can, in principle, lead to the longitudinal counterterms in higher loops and therefore it is dangerous for the consistency of our model. Indeed for the non-abelian vector fields A_μ^a which are only present in SM, the anomaly is impossible due to algebraic reasons.

It turns out that introducing *scalar field* ϕ into the fermion-torsion system is difficult if not impossible. As we already know from the study of QFT on an external torsion background, in presence of Yukawa interactions one has to introduce the "nonminimal" $S_\mu^2\phi^2$-vertex. Also in the torsion action appears additional interaction term, so it becomes

$$S_{tor} = \int d^4x\, \{-\frac{1}{4}S_{\mu\nu}^2 + M_{ts}^2 S_\mu^2 - \frac{1}{24}\zeta(S\cdot S)^4\} + ... \tag{40}$$

Here ζ is new arbitrary parameter, and coefficient $\frac{1}{24}$ stands for the sake of convenience only. These two vertices can lead to problem with consistency of all the approach.

The root of the problem is that the Yukawa interaction term $h\varphi\bar{\psi}\psi$ is not invariant under the transformation (36). Unlike the spinor mass the Yukawa constant h is massless, and therefore this noninvariance may affect the renormalization in the massless sector of the theory. In particular, the noninvariance of the Yukawa interaction causes the necessity of the nonminimal scalar-torsion interaction and the self-interaction term in (40). At one loop there are no divergent kinetic diagrams with these new vertices. But at two-loop level two such diagrams show up, they are shown at Fig. 1. Those diagrams are divergent and

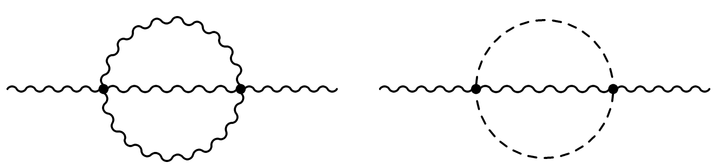

Figure 1: *The wavy line is torsion propagator and dashed line – scalar propagator*

they can lead to the appearance of the $(\partial_\mu S^\mu)^2$-type counterterm. No any symmetry is seen which forbids these divergences. Direct calculation confirms that even the $1/\epsilon^2$-pole doesn't cancel.

Let us consider two diagrams in more details. Using the actions of the scalar field coupled to torsion and the torsion self-interaction (40), we arrive at the following Feynman rules:

i) Scalar propagator: $\quad G(k) = \frac{i}{k^2+M^2}$ where $M^2 = 2M_{ts}^2$,

ii) Torsion propagator: $\quad D_\mu^\nu(k) = \frac{i}{k^2+M^2}\left(\delta_\mu^\nu + \frac{k_\mu k^\nu}{M^2}\right)$,

iii) Torsion^2–scalar^2 vertex: $\quad V^{\mu\nu}(k,p,q,r) \,=\, -\,2i\xi\,\eta^{\mu\nu}$,

iv) Self-interaction vertex: $V_{\mu\nu\alpha\beta}(k,p,q,r) \,=\, \frac{i\zeta}{3}\,g^{(2)}_{\mu\nu\alpha\beta}$

where

$$g^{(2)}_{\mu\nu\alpha\beta} = g_{\mu\nu}g_{\alpha\beta} + g_{\mu\beta}g_{\alpha\nu} + g_{\mu\alpha}g_{\nu\beta}$$

and k,p,q,r denote the outgoing momenta.

It is necessary to check the violation of the transversality in the kinetic 2-loop counterterms. It turns out that it is sufficient to trace the $\frac{1}{\epsilon^2}$-pole, because even this leading divergence requires the longitudinal counterterm. The contribution to the mass-operator of torsion from the second diagram from is given by the following integral

$$\Pi^{(2)}_{\alpha\beta}(q) = -\int \frac{d^Dk}{(2\pi)^4}\frac{d^Dp}{(2\pi)^4}\,\frac{2\,\xi^2\,\{\,\eta_{\alpha\beta}\,+\,M^{-2}\,(k-q)_\alpha\,(k-q)_\beta\,\}}{(p^2+M^2)[(k-q)^2+M^2][(p+k)^2+M^2]}\,. \tag{41}$$

First one has to notice that (as in any local quantum field theory) the counterterms needed to subtract the divergences of the above integrals are local expressions, hence the divergent part of the above integral is finite polynomial in the external momenta q^μ. In order to extract these divergences one can expand the factor in the integrand into the power series in q^μ and substitute this expansion into the integral [6]. It is easy to see that the divergences hold in this expansion till the order $n = 8$. On the other hand, each order brings some powers of q^μ. The divergences of the above integral may be canceled only by adding the counterterms which include high derivatives. This is a consequence of the power-counting nonrenormalizability of the theory with massive vector fields.

To achieve the renormalizability one has to include these high derivative terms into the action (40). However, since we are aiming to construct the effective (low-energy) field theory of torsion, the effects of the higher derivative terms are not seen and their renormalization is not interesting for us. All we need are the second derivative counterterms. Hence, for our purposes the expansion can be cut at $n = 2$ rather that at $n = 8$ and moreover only $O(q^2)$ terms should be kept. Then, using symmetry considerations, one arrives at

$$\Pi^{(2)}_{\alpha\beta}(q) = -\frac{12\,\xi^2}{(4\pi)^4\,(D-4)^2}\,q^2\,\eta_{\alpha\beta} + ...$$

Another integral looks a bit more complicated, but its derivation performs in a similar way. The result reads [6]

$$\Pi^{(1)}_{\alpha\lambda}(q) = -\frac{\zeta^2}{(4\pi)^4\,(D-4)^2}\,q^2\,\eta_{\alpha\lambda} +\ \dots\,. \tag{42}$$

Thus we see that both diagrams really give rise to the longitudinal kinetic counterterm and no any simple cancellation of these divergences is seen. On the other hand one can hope to achieve such a cancellation on the basis of some sofisticated symmetry.

Let us compare the torsion theory with the the usual abelian gauge theory. In this case the symmetry is not violated by the Yukawa coupling, and (in the abelian case) the $A^2\varphi^2$ counterterm is impossible. The same concerns also the self-interacting A^4 counterterm. The gauge invariance of the theory on quantum level is controlled by the Ward identities. In the theory of torsion field coupled to the MSM with scalar field there are no reasonable gauge identities at all. So there is a conflict between renormalizability and unitarity, which reminds the one which is well known – the problem of massive unphysical ghosts in the high derivative gravity. The difference is that in our case, unlike higher derivative gravity, the problem appears due to the non invariance with respect to the transformation (36).

Let us now discuss how this problem may be, in principle, solved.

i) If the torsion mass is of the Planck order then the quantum effects of torsion should be described directly in the framework of string theory. No any effective field theory for torsion is possible. In this case the only visible term in the torsion action is the massive one and torsion does not propagate at smaller energies.

ii) There may be a hope to impose one more symmetry which is not violated by the Yukawa coupling. It can be, for example, supersymmetry which mixes torsion with some vector fields of the SM and with all massive spinor fields. In this case the $(\partial_\mu S^\mu)^2$-type counterterm may be forbidden by this symmetry and the conflict between renormalizability and unitarity would be resolved.

iii) Consider the modification of SM which is free from the fundamental scalar fields.

Let us briefly discuss the *renormalization group in the theory with torsion.* We consider the spinor-torsion system with an additional electromagnetic field, but without the controversial scalar. Then the renormalization group equations for the parameters e, η, m, M_{ts}

$$(4\pi)^2\,\frac{de}{dt} = \frac{2}{3}\,e^2\,, \qquad e(0) = e_0$$

$$(4\pi)^2\,\frac{d\eta}{dt} = \frac{2}{3}\,\eta^2\,, \qquad \eta(0) = \eta_0$$

$$(4\pi)^2\,\frac{dM_{ts}^2}{dt} = 8\,m^2\,\eta^2 - 2\,M_{ts}^2\,, \qquad M_{ts}(0) = M_{ts,0}\,. \tag{43}$$

We remark that the last equation demonstrates the inconsistency of the massless or very light torsion. Even if one imposes the normalization condition $M_{ts,0} \approx 0$ at some scale μ, the first term in this equation provides a rapid change of M_{ts} such that it will be essentially nonzero at other scales. Due to the universality of the interaction with torsion all quarks and massive leptons should contribute to this equation. Therefore the only way to avoid an unnaturally fast running of M_{ts} is to take its value at least of the order of the heaviest spinor field that is t-quark. Hence we have some grounds to take $M_{ts} \geq 100\,\mathrm{GeV}$. Of course there can not be any upper bounds for M_{ts} from the RG equation.

Phenomenology of propagating torsion.

The spinor-torsion interactions enter the Standard Model as interactions of fermions with new axial vector field S_μ. Such an interaction is characterized by the new dimensionless parameter – coupling constant η. Furthermore the mass of the torsion field M_{ts} is unknown, and its value is of crucial importance for the possible experimental manifestations of the propagating torsion and finally for the existence of torsion at all. Below we consider η and M_{ts} as an arbitrary parameters and try to limit their values from the known experiments. Indeed we use the renormalization group as an insight concerning the mass of torsion but include the discussion of the "light" torsion with the mass of the order of 1 GeV for the sake of generality.

Our strategy will be to use known experiments directed to the search of the new interactions. We regard torsion as one of those interactions and obtain the limits for the torsion parameters from the data which already fit with the phenomenological considerations [6].

Therefore we insert torsion into the minimal SM and suppose that the other possible new physics is absent. It is common assumption when one wants to put limits on some particular kind of a new physics. In this way one can put the limits on the parameters of the torsion action using results of various experiments. We refer the reader to the original work [6] for the full details, and here present just a brief review.

It is reasonable to consider two different cases:

i) Torsion is much more heavy than other particles of SM

ii) Torsion has a mass comparable to that of other particles. In the last case one meets a propagating particle which must be treated on an equal footing with other constituents of the SM.

Indeed the very heavy torsion leads to the effective contact four-fermion interactions. Consider the case of heavy torsion in some more details. Since the massive term dominates over the covariant kinetic part of the action, the last can be disregarded. Then the total action leads to the algebraic equation of motion for S_μ. The solution of this equation can be substituted back to $S_{1/2} + S_{tor}$ and thus produce the contact four-fermion interaction term

$$\mathcal{L}_{int} = -\frac{\eta^2}{M_{ts}^2}\,(\bar{\psi}\gamma_5\gamma^\mu\psi)\,(\bar{\psi}\gamma_5\gamma_\mu\psi) \tag{44}$$

As one can see the only one quantity which appears in this approach is the ratio M_{ts}/η and therefore for the very heavy torsion field the phenomenological consequences depend only on single parameter.

Physical observables related with torsion depend on the two parameters M_{ts} and η. In the course of our study we choose, for the sake of simplicity, all the torsion couplings with fermions to be the same η. This enables one to put the limits in the two dimensional $M_{ts} - \eta$ parameter space using the present experimental data. We also assume that non-diagonal coupling of the torsion with the fermions of different families is zero in order to avoid flavor changing neutral current problem.

Another possibility comes from the consideration of the the axial-vector type interactions would give rise to the forward-backward asymmetry. This asymmetry has been presizely measured in the $e^+e^- \to l^+l^-(q\bar{q})$ scattering (here l stands for tau,muon or electron) at LEP collider with the center-mass energy equals to the Z-boson mass. The corresponding diagrams are shown at Fig. 2.

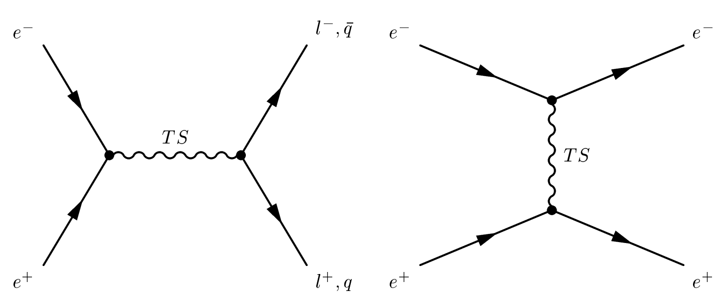

Figure 2: *Diagrams with torsion exchange in e^+e^- collisions contributing to forward-backward lepton and quark asymmetry. TS indicates to the torsion propagator.*

Indeed any parity violating interactions give rise to the space asymmetry and could be reviled in a forward-backward asymmetry measurement. Axial-vector type interactions of torsion with matter fields is this case of interactions. But the source of asymmetry also exists electroweak interactions because of the presence of the $\gamma_\mu\gamma_5$ structure in the interactions of Z- and W-bosons with fermions. Such an asymmetries are measured at LEP. Presence of torsion would change the forward-backward asymmetry and would brightly reviel itself since its contribution differs from the one of other fields [6]. One has to notice the measured EW parameters are in a good agreement with the theoretical predictions and hence the limits established on the torsion parameters are based on the experimental errors.

The straightforward consequence of the heavy torsion interacting with fermion fields is the effective four-fermion contact interaction of leptons and quarks. The contact interactions were widely discussed in 70th (see [7] for the review.) Four-fermion interaction effectively appears for the torsion with a mass much higher than the energy scale available at present colliders. Thus it is interesting to discuss the possibility for detecting torsion effects in the framework of new experimental possibilities. There are several experiments from which the constraints on the contact four-fermion interactions come. One can find further details in [6]. The specific distinguishing feature of the contact interactions induced by torsion is that those contact interactions are of axial-axial type. In [6] we have used the limits obtained by other authors for this kind of interaction.

The torsion with the mass in the range of present colliders could be produced in fermion-fermion interactions as a resonance, decaying to fermion pair. The most promising collider for search the signature of such type is TEVATRON. The total limits on torsion parameters coming from all mentioned experiments, can be found in [6].

Conclusion and Acknowledgments.

We have reviewed the recent developments about the possible manifestations of torsion field. In particular, the non-relativistic approximation to the Dirac equation and the corresponding action of particle have been derived. Also we discussed the possibility to implement propagating torsion into the Standard Model of the elementary particle physics, and found that this can be done, but only for the fermion sector of the SM. The introduction of torsion into the full SM including Higgs fields meets serious difficulties.

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